

FUZZY DOT SUBALGEBRA AND FUZZY DOT i^* -IDEAL OF i^* -ALGEBRAP. Manjula¹ and P. Sundararajan²¹Research Scholar, Department of Mathematics, Periyar University, Salem, Tamil Nadu, India.²Associate Professor, PG & Research Department of Mathematics, Arignar Anna Govt. Arts College, Namakkal. Tamil Nadu, India.

Abstract: In this paper a new type of i^* -algebra has been defined. Also, fuzzy dot subalgebra and fuzzy dot i^* -ideal of i^* -algebra introduced and its properties are derived.

Keywords: i^* -algebra, i^* -ideal, fuzzy dot subalgebra, fuzzy dot i^* -ideal.

1. Introduction

According to Zadeh [8] is introduced to the idea of fuzzy sets (Fs). Since then, a variety of algebraic structures, such as semigroups, groups, rings, modules, vector spaces, and topologies, have been studied using these concepts. Fs were introduced by Xi [7] in 1991 and were later used to the BCK -algebras suggested by Imai and Iséki [2,3].

In this paper, we describe the concept of a fuzzy dot subalgebra of an i^* -algebra as an extension of a fuzzy subalgebra of an i^* -algebra. Following that, we'll examine at some of the essential properties of fuzzy dot subalgebras. We also examine the concept of fuzzy dot i^* -algebraic ideals before delving into several essential properties associated with these ideals. In section 2 describe the basic preliminaries and Fuzzy dot subalgebra (FDs) of i^* -algebra discussed in section 3. In section 4 proved some theorem and definition with example of Fuzzy dot i^* -ideal of i^* -algebra. Finally Conclusion given in the last section.

2. Preliminaries

2.1 Definition: Cartesian product of fuzzy sets

Let ψ and ϑ be the Fs in a set Θ . The Cartesian product $(C_p) \psi \times \vartheta: \Theta \times \Theta \rightarrow [0, 1]$ is defined by $(\psi \times \vartheta)(\theta, y) = \psi(\theta) \cdot \vartheta(y)$, for all $\theta, y \in \Theta$.

2.2 Example

Define a fuzzy set ψ in $\Theta = \{0, 1, 2\}$ by $\psi(0) = 0.1$, $\psi(1) = 0.2$, $\psi(2) = 0.5$. Define a fuzzy set ϑ in Θ by $\vartheta(0) = 1$, $\vartheta(1) = 0.5$, $\vartheta(2) = 0.2$.

Then the Cartesian product $\psi \times \vartheta: \Theta \times \Theta \rightarrow [0, 1]$ is defined as $(\psi \times \vartheta)(0, 1) = \psi(0) \cdot \vartheta(1) = (0.1) \cdot (0.5) = 0.05$. Similarly we can get values for other elements in $\Theta \times \Theta$.

2.3 Definition: Union of fuzzy subsets

For any fuzzy subsets (FSs) ϑ and γ of a set Θ , we define $(\vartheta \cup \gamma)(\theta) = \max\{\vartheta(\theta), \gamma(\theta)\}$ for all $\theta \in \Theta$.

2.4 Example

Let F_s, ϑ in $\Theta = \{0, 1, 2\}$ by $\vartheta(0) = 0.1, \vartheta(1) = 0.2, \vartheta(2) = 0.5$.

Let F_s, γ in Θ by $\gamma(0) = 1, \gamma(1) = 0.5, \gamma(2) = 0.2$.

$$\Rightarrow (\vartheta \cup \gamma)(0) = \max\{\vartheta(0), \gamma(0)\} = \max\{0.1, 1\} = 1.$$

Similarly we can definitely for other elements in Θ .

3. Fuzzy dot subalgebra (FDs) of i^* -algebra

3.1 Definition: i^* -algebra

A i^* -algebra is an algebra $(\Theta, *, 0)$ of type $(2, 0)$ satisfying the following conditions.

(i) $\theta * \theta = 1, \theta \neq 0 \in \Theta$

(ii) $0 * \theta = 0, \theta \neq 0 \in \Theta$

(iii) $\theta * y = 1$ and $y * \theta = 1$

$\Rightarrow \theta = y$ for all $\theta \neq 0, y \neq 0 \in \Theta$.

3.2 Example

Let $\Theta = \{0, a, b, \}$. Consider the following Cayley table

*	0	a	b
0	1	0	0
a	0	1	0
b	0	1	1

Table 1: the system $(\Theta, *, 0)$ is a i^* -algebra

Clearly this table shows that, the system $(\Theta, *, 0)$ is a i^* -algebra.

3.3 Definition: Subalgebra of I^* -algebra

Let S be a nonempty subset of a i^* -algebra Θ . Then S is called a subalgebra of Θ if $\theta * y \in S$, for all $\theta, y \in S$.

3.4 Example

Consider a i^* -algebra $\Theta = \{0, 1, 2\}$ having the following Cayley table.

*	0	1	2
0	1	0	0
1	0	1	2
2	0	1	1

Table 2: $S = \{0, 1\}$ is a subalgebra of a i^* -algebra

Clearly the table 2 shows that $S = \{0, 1\}$ is a subalgebra of a i^* -algebra.

3.5 Definition: *fuzzy dot sub algebra of a i^* -algebra*

A fuzzy subset ϑ of Θ is called fuzzy dot sub algebra of a i^* -algebra Θ if $\vartheta(\theta*y) \leq \vartheta(\theta) \cdot \vartheta(y)$ for all $\theta, y, \theta = y \neq 1 \in \Theta$.

3.6 Example

Consider a i^* -algebra $\Theta = \{0,1,2\}$ having the following Cayley table:

*	0	1	2
0	1	0	0
1	0	1	1
2	0	0	1

Define a fuzzy set ϑ in Θ by $\vartheta(0)=1, \vartheta(1)=0.2, \vartheta(2)=1$. It is easy to verify that ϑ is a fuzzy dot subalgebra of a i^* -algebra.

3.7 Theorem

If ϑ is a fuzzy dot subalgebra of a i^* -algebra Θ , then we have $\vartheta(1) \leq (\vartheta(\theta))^2$ for all $\theta \neq 1 \in \Theta$.

Proof

For every $\theta \neq 1 \in \Theta$,

we have, $\vartheta(1) = \vartheta(\theta*\theta) \leq \vartheta(\theta) \cdot \vartheta(\theta) = (\vartheta(\theta))^2$

This completing the Proof:

3.8 Theorem

If ϑ and γ are fuzzy dot subalgebra of a i^* -algebra Θ , then so is $\vartheta \cup \gamma$.

Proof

Let $\theta, y \in \Theta$.

$$\begin{aligned} \text{Then } (\vartheta \cup \gamma)(\theta * y) &= \text{ma}\theta \{ \vartheta(\theta*y), \gamma(\theta*y) \} \\ &\leq \text{ma}\theta \{ \vartheta(\theta) \cdot \vartheta(y), \gamma(\theta) \cdot \gamma(y) \} \\ &\leq (\text{ma}\theta \{ \vartheta(\theta), \gamma(\theta) \}) \cdot (\text{ma}\theta \{ \vartheta(y), \gamma(y) \}) \\ &= ((\vartheta \cup \gamma)(\theta)) \cdot ((\vartheta \cup \gamma)(y)). \end{aligned}$$

Hence $\vartheta \cup \gamma$ is a fuzzy dot subalgebra of a i^* -algebra Θ .

3.9 Theorem

If ψ and ϑ are fuzzy dot subalgebras of a i^* -algebra Θ , then $\psi \times \vartheta$ is a fuzzy dot subalgebra of $\Theta \times \Theta$.

Proof

For any $\theta_1, \theta_2, y_1, y_2 \in \Theta$,

$$\begin{aligned} (\psi \times \vartheta)((\theta_1, y_1) * (\theta_2, y_2)) &= (\psi \times \vartheta)(\theta_1 * \theta_2, y_1 * y_2) \\ &= \psi(\theta_1 * \theta_2) \cdot \vartheta(y_1 * y_2) \\ &\leq ((\psi(\theta_1) \cdot \psi(\theta_2)) \cdot ((\vartheta(y_1) \cdot \vartheta(y_2))) \\ &= ((\psi(\theta_1) \cdot \vartheta(y_1)) \cdot (\psi(\theta_2) \cdot \vartheta(y_2))) \\ &= (\psi \times \vartheta)(\theta_1, y_1) \cdot (\psi \times \vartheta)(\theta_2, y_2), \end{aligned}$$

This completing the Proof:

3.10 Definition: *strongest fuzzy ϕ -relation on i^* -algebra*

The strongest fuzzy ϕ -relation on i^* -algebra Θ is the fuzzy subset \mathcal{G}_ϕ of $\Theta \times \Theta$ given by $\mathcal{G}_\phi(\theta, y) = \phi(\theta) \cdot \phi(y)$ for all $\theta, y \in \Theta$.

3.11 Definition: *fuzzy ϕ -product relation*

A fuzzy relation \mathcal{G} on i^* -algebra Θ is called a fuzzy ϕ -product relation if $\mathcal{G}(\theta, y) \leq \phi(\theta) \cdot \phi(y)$ for all $\theta, y \in \Theta$.

3.12 Definition: *left fuzzy relation*

A fuzzy relation \mathcal{G} on i^* -algebra Θ is called a left fuzzy relation on ϕ if $\mathcal{G}(\theta, y) = \phi(\theta)$ for all $\theta, y \in \Theta$. Note that a left fuzzy relation on ϕ is a fuzzy ϕ -product relation.

3.13 Theorem

Let \mathcal{G}_ϕ be the strongest fuzzy ϕ -relation on i^* -algebra Θ , where ϕ is a fuzzy subset of a i^* -algebra Θ . If ϕ is a fuzzy dot subalgebra of a i^* -algebra Θ , then \mathcal{G}_ϕ is a fuzzy dot subalgebra of $\Theta \times \Theta$.

Proof

Suppose that ϕ is fuzzy dot subalgebra of Θ .

For any $\theta_1, \theta_2, y_1, y_2 \in \Theta$, we have,

$$\begin{aligned} \mathcal{G}_\phi((\theta_1, y_1) * (\theta_2, y_2)) &= \mathcal{G}_\phi(\theta_1 * \theta_2, y_1 * y_2) \\ &= \phi(\theta_1 * \theta_2) \cdot \phi(y_1 * y_2) \\ &\leq \\ (\phi(\theta_1) \cdot \phi(\theta_2)) \cdot (\phi(y_1) \cdot \phi(y_2)) \\ &= (\phi(\theta_1) \cdot \phi(y_1)) \cdot (\phi(\theta_2) \cdot \phi(y_2)) \\ &= \mathcal{G}_\phi(\theta_1, y_1) \cdot \mathcal{G}_\phi(\theta_2, y_2), \end{aligned}$$

so \mathcal{G}_ϕ is a fuzzy dot subalgebra of $\Theta \times \Theta$.

3.14 Theorem

Let \mathcal{G} be a left fuzzy relation on a fuzzy subset ϕ of a i^* -algebra Θ . If \mathcal{G} is a fuzzy dot subalgebra of $\Theta \times \Theta$, then ϕ is a fuzzy dot subalgebra of a i^* -algebra Θ .

Proof

Suppose that a left fuzzy relation \mathcal{G} on ϕ is fuzzy dot subalgebra of $\Theta \times \Theta$.

$$\begin{aligned} \text{Then } \phi(\theta_1 * \theta_2) &= \mathcal{G}(\theta_1 * \theta_2, y_1 * y_2) \\ &= \mathcal{G}((\theta_1, y_1) * (\theta_2, y_2)) \\ &\leq \mathcal{G}(\theta_1, y_1) \cdot \mathcal{G}(\theta_2, y_2) \\ &= \phi(\theta_1) \cdot \phi(\theta_2) \text{ for all } \theta_1, \theta_2, y_1, y_2 \in \Theta. \end{aligned}$$

Hence ϕ is a fuzzy dot subalgebra of a i^* -algebra Θ .

4. Fuzzy dot i^* -ideal of i^* -algebra**4.1 Definition:** *i^* -ideal of i^* -algebra*

Let Θ be a i^* -algebra and I be a subset of Θ , then I is called i^* -ideal of Θ if it satisfies the following conditions.

- (i) $1 \in I$
- (ii) $\theta * y \in I$ and $y \in I \Rightarrow \theta \in I$

$$(iii) \quad \theta \in I, y \in \Theta \\ \Rightarrow \theta * y \in I.$$

4.2 Example:

Consider a i^* -algebra $\Theta = \{0, 1, 2, 3\}$ having the following Cayley table:

*	0	1	2	3
0	0	0	0	0
1	1	1	2	0
2	2	1	1	0
3	1	0	0	1

Table 4: $I = \{0, 1, 2\}$ is i^* -ideal of i^* -algebra

Clearly this table shows that $I = \{0, 1, 2\}$ is i^* -ideal of i^* -algebra.

4.3 Definition: *fuzzy i^* -ideal i^* -algebras*

A fuzzy subset μ of Θ is called a fuzzy i^* -ideal of Θ if it satisfies the following conditions for all $\theta, y \in \Theta$:

- (i) $\vartheta(1) \leq \vartheta(\theta)$
- (ii) $\vartheta(\theta) \leq \max\{\vartheta(\theta * y), \vartheta(y)\}$
- (iii) $\vartheta(\theta * y) \leq \max\{\vartheta(\theta), \vartheta(y)\}$

4.4 Example

For the table 3, Define a fuzzy set ϑ in Θ by $\vartheta(0)=0.6, \vartheta(1)=0.5, \vartheta(2)=0.8, \vartheta(3)=0.9$. Then it is easy to verify that ϑ is a fuzzy i^* -ideal algebra of i^* -algebra Θ .

4.5 Definition: *Fuzzy Dot i^* -ideals of i^* -algebras*

A fuzzy subset μ of Θ is called a fuzzy dot i^* -ideal of Θ if it satisfies the following conditions for all $\theta, y, \theta = y \neq 1 \in \Theta$.

- (i) $\vartheta(1) \leq \vartheta(\theta)$
- (ii) $\vartheta(\theta) \leq \vartheta(\theta * y) \cdot \vartheta(y)$
- (iii) $\vartheta(\theta * y) \leq \vartheta(\theta) \cdot \vartheta(y)$

4.6 Example

Let $\Theta = \{0, 1, a, b\}$. Consider the following Cayley table.

*	0	a	b	1
0	0	0	0	0
a	a	1	0	a
b	b	0	1	b
1	1	0	0	1

Table 5: $(\Theta, *, 0)$ is a i^* -algebra. $I = \{0, a, 1\}$ is a i^* -ideal

Clearly this table shows that, the system $(\Theta, *, 0)$ is a i^* -algebra. $I = \{0, a, 1\}$ is a i^* -ideal.

Define a fuzzy set ϑ in Θ by $\vartheta(0)=1, \vartheta(1)=0.5, \vartheta(a)=\vartheta(b)=1$. Then it is easy to verify that ϑ is a fuzzy dot subalgebra of a i^* -algebra. Also it is fuzzy dot i^* -ideal of i^* -algebra Θ .

4.7 Theorem

Every fuzzy dot i^* -ideal of a i^* -algebra Θ is a fuzzy dot subalgebra of Θ .

Proof

By the definition of fuzzy dot i^* -ideal of a i^* -algebra Θ , it is clearly true that every fuzzy dot i^* -ideal of a i^* -algebra Θ is a fuzzy dot subalgebra of Θ .

4.8 Remark

The converse of Proposition is not true.

4.9 Theorem

If ϑ and γ are fuzzy dot i^* -ideals of a i^* -algebra Θ , then so is $\vartheta \cup \gamma$.

Proof

Let $\theta, y \in \Theta$.

(I) Then $(\vartheta \cup \gamma)(1) = \text{ma}\theta\{\vartheta(1), \gamma(1)\}$

$$\leq \text{ma}\theta\{\vartheta(\theta), \gamma(\theta)\}$$

$$= (\vartheta \cup \gamma)(\theta).$$

(ii) Also, $(\vartheta \cup \gamma)(\theta) = \text{ma}\theta\{\vartheta(\theta), \gamma(\theta)\}$

$$\leq \text{ma}\theta\{\vartheta(\theta * y) \cdot \vartheta(y), \gamma(\theta * y) \cdot \gamma(y)\}$$

$$\leq (\text{ma}\theta\{\vartheta(\theta * y), \gamma(\theta * y)\}) \cdot (\text{ma}\theta\{\vartheta(y), \gamma(y)\})$$

$$= ((\vartheta \cup \gamma)(\theta * y)) \cdot ((\vartheta \cup \gamma)(y)).$$

(iii) And, $(\vartheta \cup \gamma)(\theta * y) = \text{ma}\theta\{\vartheta(\theta * y), \gamma(\theta * y)\}$

$$\leq \text{ma}\theta\{\vartheta(\theta) \cdot \vartheta(y), \gamma(\theta) \cdot \gamma(y)\}$$

$$\leq (\text{ma}\theta\{\vartheta(\theta), \gamma(\theta)\}) \cdot (\text{ma}\theta\{\vartheta(y), \gamma(y)\})$$

$$= ((\vartheta \cup \gamma)(\theta)) \cdot ((\vartheta \cup \gamma)(y)).$$

Hence $\vartheta \cup \gamma$ is a fuzzy dot i^* -ideal of a i^* -algebra Θ .

4.10 Theorem

If ψ and ϑ are fuzzy dot i^* -ideal of a d-algebra Θ , then $\psi \times \vartheta$ is a fuzzy dot i^* -ideal of $\Theta \times \Theta$.

Proof

Let $\theta, y \in \Theta$.

$$(i) \quad (\psi \times \vartheta)(1, 1) = \psi(1) \cdot \vartheta(1) \leq \psi(\theta) \cdot \vartheta(y)$$

For any $\theta, \theta_1, y, y_1 \in \Theta$, we have

$$(ii) \quad (\psi \times \vartheta)(\theta, y) = \psi(\theta) \cdot \vartheta(y) \\ \leq (\psi(\theta * \theta_1) \cdot \psi(\theta_1)) \cdot (\vartheta(y * y_1) \cdot \vartheta(y_1)) \\ = (\psi(\theta * \theta_1) \cdot \vartheta(y * y_1)) \cdot (\psi(\theta_1) \cdot \vartheta(y_1)) \\ = (\psi \times \vartheta)(\theta, y) \cdot (\psi \times \vartheta)(\theta_1, y_1)$$

$$(iii) \quad (\psi \times \vartheta)((\theta, y) * (\theta_1, y_1)) = (\psi \times \vartheta)((\theta * \theta_1), (y, y_1)) \\ = \psi(\theta * \theta_1) \cdot \vartheta(y * y_1) \\ \leq ((\psi(\theta) \cdot \psi(\theta_1)) \cdot (\vartheta(y) \cdot \vartheta(y_1))) \\ = (\psi(\theta) \cdot \vartheta(y)) \cdot (\psi(\theta_1) \cdot \vartheta(y_1)) \\ = (\psi \times \vartheta)(\theta, y) * (\psi \times \vartheta)(\theta_1, y_1)$$

Hence $\psi \times \vartheta$ is a fuzzy dot d-ideal of $\Theta \times \Theta$.

4.11 Theorem

Let ϕ be a fuzzy subset of a i^* -algebra Θ and ϕ_ϑ be the strongest fuzzy ϕ -relation on i^* -algebra Θ . Then ϕ is a fuzzy dot i^* -ideal of Θ if and only if ϕ_ϑ is a fuzzy dot i^* -ideal of $\Theta \times \Theta$.

Proof

Assume that ϕ is a fuzzy dot d-ideal of Θ .

$$\text{For any } \theta, y \in \Theta \text{ we have } \vartheta_\phi(1, 1) = \phi(1) \cdot \phi(1) \\ \leq \phi(\theta) \cdot \phi(y) \\ = \vartheta_\phi(\theta, y).$$

Let $\theta, \theta', y, y' \in \Theta$.

$$\text{Then } \vartheta_\phi((\theta, \theta') * (y, y')) \cdot \vartheta_\phi(y, y') = \vartheta_\phi(\theta * y, \theta' * y') \cdot \vartheta_\phi(y, y') \\ = (\phi(\theta * y) \cdot \phi(\theta' * y')) \cdot (\phi(y) \cdot \phi(y')) \\ = (\phi(\theta * y) \cdot \phi(y)) \cdot (\phi(\theta' * y') \cdot \phi(y')) \\ \geq \phi(\theta) \cdot \phi(\theta') \\ = \vartheta_\phi(\theta, \theta').$$

$$\text{and, } \vartheta_\phi(\theta, \theta') \cdot \vartheta_\phi(y, y') = (\phi(\theta) \cdot \phi(\theta')) \cdot (\phi(y) \cdot \phi(y')) \\ = (\phi(\theta) \cdot \phi(y)) \cdot (\phi(\theta') \cdot \phi(y')) \\ \geq \phi(\theta * y) \cdot \phi(\theta' * y') \\ = \vartheta_\phi(\theta * y, \theta' * y') \\ = \vartheta_\phi((\theta, \theta') * (y, y')).$$

Thus ϑ_ϕ is a fuzzy dot i^* -ideal of $\Theta \times \Theta$.

Conversely suppose that \mathfrak{g}_ϕ is a fuzzy dot i^* -ideal of $\Theta \times \Theta$.

$$\begin{aligned}(\phi(1))^2 &= \phi(1) \cdot \phi(1) = \mathfrak{g}_\phi(1,1) \leq \mathfrak{g}_\phi(\theta, \theta) \\ &= \mathfrak{g}(\theta) \cdot \mathfrak{g}(\theta) = (\mathfrak{g}(\theta))^2\end{aligned}$$

and so $\phi(1) \leq \phi(\theta)$ for all $\theta \in \Theta$. Also we have

$$\begin{aligned}(\phi(\theta))^2 &= \mathfrak{g}_\phi(\theta, \theta) \\ &\leq \mathfrak{g}_\phi((\theta, \theta) * (y, y)) \cdot \mathfrak{g}_\phi(y, y) \\ &= \mathfrak{g}_\phi((\theta * y), (\theta * y)) \cdot \mathfrak{g}_\phi(y, y) \\ &= \phi((\theta * y)) \cdot \phi(y)^2\end{aligned}$$

which implies that $\phi(\theta) \leq \phi(\theta * y) \cdot \phi(y)$ for all $\theta, y \in \Theta$. Also we have

$$\begin{aligned}(\phi(\theta * y))^2 &= \mathfrak{g}_\phi(\theta * y, \theta * y) \\ &= \mathfrak{g}_\phi((\theta, \theta) * (y, y)) \\ &\leq \mathfrak{g}_\phi(\theta, \theta) \cdot \mathfrak{g}_\phi(y, y) \\ &= (\phi(\theta) \cdot \phi(y))^2.\end{aligned}$$

So $\phi(\theta * y) \leq \phi(\theta) \cdot \phi(y)$ for all $\theta, y \in \Theta$. Therefore ϕ is a fuzzy dot i^* -ideal of Θ .

4.12 Theorem

Let \mathfrak{g} be a left fuzzy relation on a fuzzy subset ϕ of a i^* -algebra Θ . If \mathfrak{g} is a fuzzy dot d -ideal of $\Theta \times \Theta$, then ϕ is a fuzzy dot i^* -ideal of a i^* -algebra Θ .

Proof

Suppose that a left fuzzy relation \mathfrak{g} on ϕ is a fuzzy dot i^* -ideal of $\Theta \times \Theta$.

Then $\phi(1) = \mathfrak{g}(1, z)$, $\forall z \in \Theta$.

By putting $z=1$

$$\phi(1) = \mathfrak{g}(1, 1) \leq \mathfrak{g}(\theta, y) = \phi(\theta), \text{ for all } \theta \in \Theta.$$

For any $\theta, \theta', y, y' \in \Theta$

$$\begin{aligned}\phi(\theta) &= \mathfrak{g}(\theta, y) \leq \mathfrak{g}((\theta, y) * (\theta', y')) \cdot \mathfrak{g}(\theta', y') \\ &= \mathfrak{g}((\theta * \theta'), (y * y')) \cdot \mathfrak{g}(\theta', y') \\ &= \phi(\theta * \theta') \cdot \phi(\theta').\end{aligned}$$

Also, $\phi(\theta * \theta') = \mathfrak{g}(\theta * \theta', y * y')$

$$\begin{aligned}&= \mathfrak{g}((\theta, y) * (\theta', y')) \\ &\leq \mathfrak{g}(\theta, y) \cdot \mathfrak{g}(\theta', y') \\ &= \phi(\theta) \cdot \phi(\theta').\end{aligned}$$

Thus ϕ is a fuzzy dot i^* -ideal of a i^* -algebra Θ .

5. Conclusion

In this paper a new type of i^* -algebra has been defined. Also fuzzy dot subalgebra and fuzzy dot i^* -ideal of i^* -algebra introduced and its properties are derived successfully. The study is still going on for the further innovative results in this i^* -algebra.

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